Lec 8 : DSP

[1] Radix-2 DIT-FFT

① $W_N^{K+N} = W_N^K$

② $W_N^{K+\frac{N}{2}} = -W_N^K$

③ $W_N^2 = W_{N/2}$

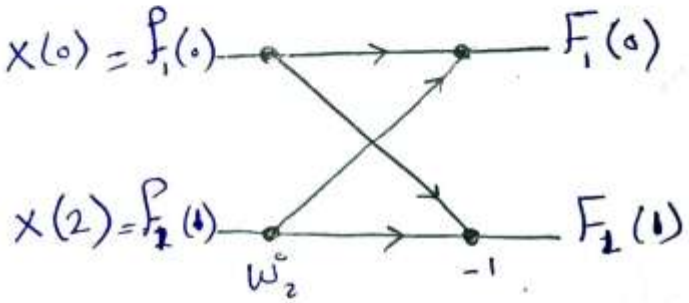
for $N = 4$ $\quad x(n) = \{x(0), x(1), x(2), x(3)\}$

← نقسمها لجزء فردي وجزء زوجي .

1)
$f_1(n) = \{x(0), x(2)\} = x(2n)$

{even numbered sequence}

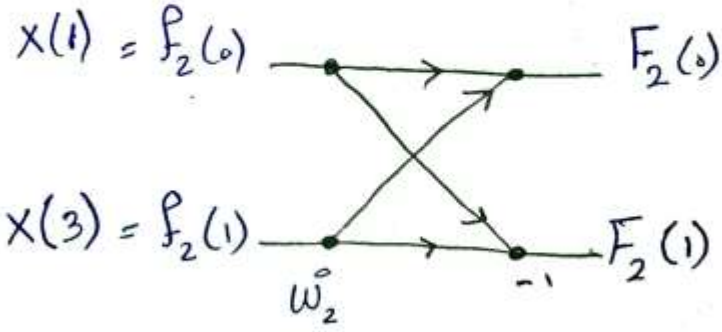
$F_1(K) \Rightarrow$ 2-Point DFT



2) $f_2(n) = \{x(1), x(3)\}$

"odd numbered sequence"

$F_2(K) \Rightarrow$ 2-Point DFT



$$① \; X(K) = F_1(K) + W_N^K \; F_2(K)$$

$$② \; X\left(K + \frac{N}{2}\right) = F_1(K) - W_N^K \; F_2(K)$$

; $K = 0, 1, 2, \ldots$

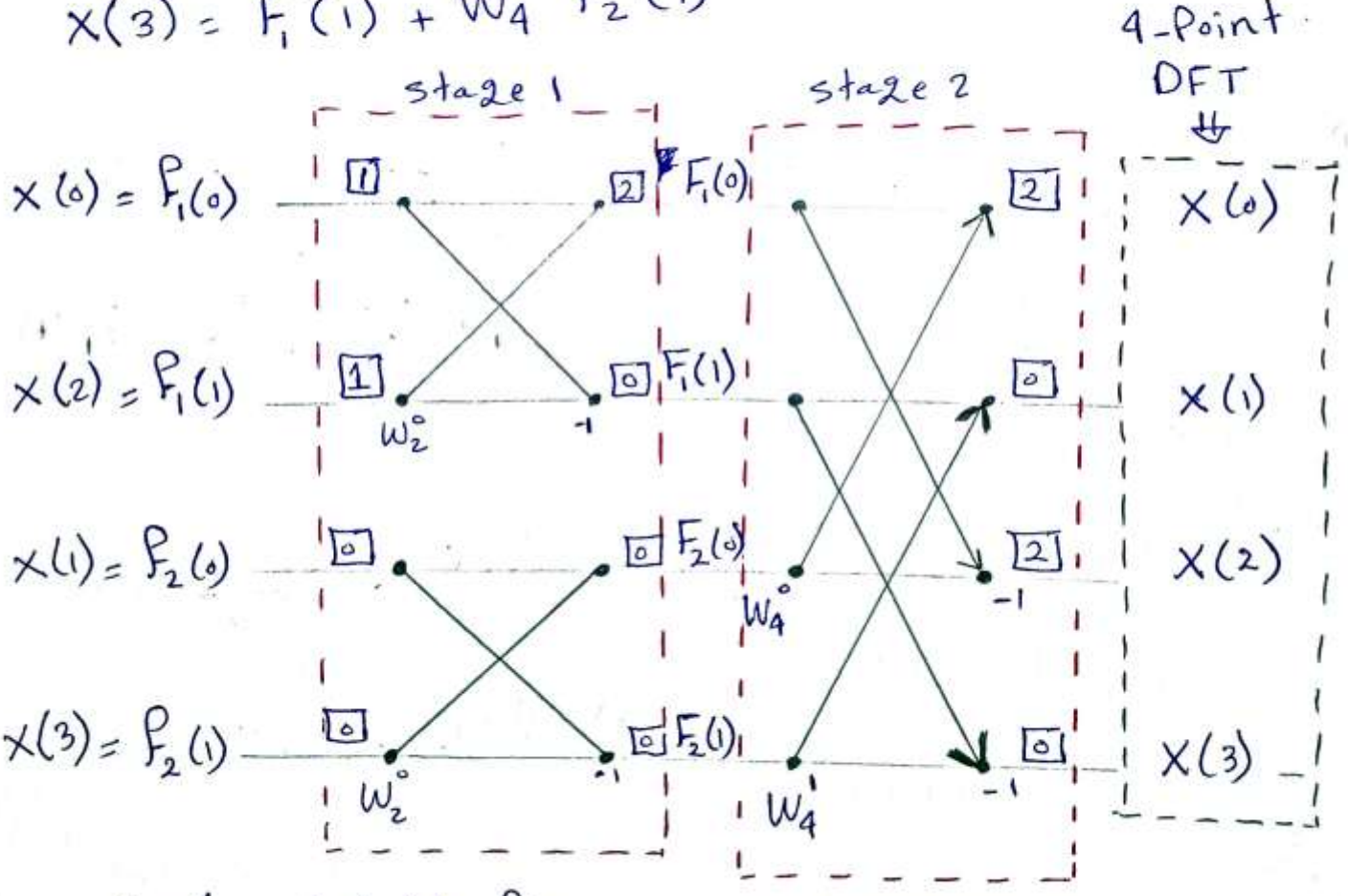
بالتعويض عن K بـ {0, 1} في كل معادلة ① ، ②

① $X(0) = F_1(0) + W_4^0 \; F_2(0)$

② $X(1) = F_1(1) + W_4^1 \; F_2(1)$

② $X(2) = F_1(0) + W_4^0 \; F_2(0)$

$X(3) = F_1(1) + W_4^1 \; F_2(1)$



N = 4 J Butter fly

الشكل العام J

← في الصفحة السابقة الرسم يعبر عن

"The signal graph with Butterfly Computation for $N=4$"

Ex: $x(n) = \{1,0,1,0\} \Rightarrow x(K) = \{2,0,2,0\}$

← الجزء بتاع $x(K)$ في حسابة في الرسم بالتحديد في (stage 2)

For $N=8$

$f_1(n) = \{x(0), x(2), x(4), x(6)\}$ ⇝ even numbered sequence

$f_2(n) = \{x(1), x(3), x(5), x(7)\}$

⇓

① $x(K) = F_1(K) + W_N^K F_2(K)$

② $x(K + \frac{N}{2}) = F_1(K) - W_N^K F_2(K)$

; $K = 0,1,2,3 \hookrightarrow \frac{N}{2} - 1$

← بالتعويض بقيم K في المعادلتين

For ①

$$x(0) = F_1(0) + W_8^0 F_2(0)$$
$$x(1) = F_1(1) + W_8^1 F_2(1)$$
$$x(2) = F_1(2) + W_8^2 F_2(2)$$
$$x(3) = F_1(3) + W_8^3 F_2(3)$$

$K = 0,1,2,3$

For eq2

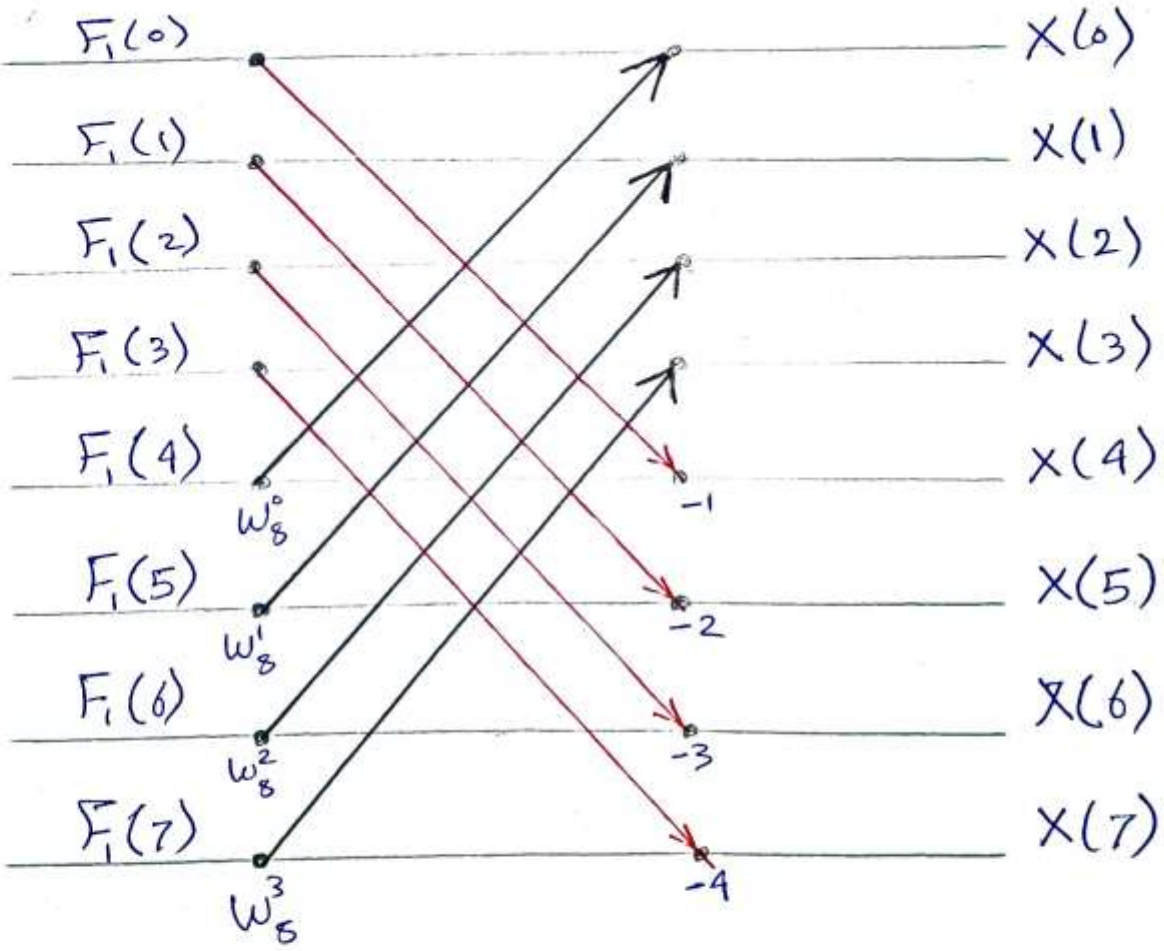
$$X(4) = F_1(0) - W_8^0 F_2(0)$$

$$X(5) = F_1(1) - W_8^1 F_2(1)$$

$$X(6) = F_1(2) - W_8^2 F_2(2)$$

$$X(7) = F_1(3) - W_8^3 F_2(3)$$

د. تكرار لـ eqn.1
لكن بإشارة سالبة



$$x(n) = \{x(0), x(1), x(2), x(3), x(4), x(5), x(6), x(7)\}$$

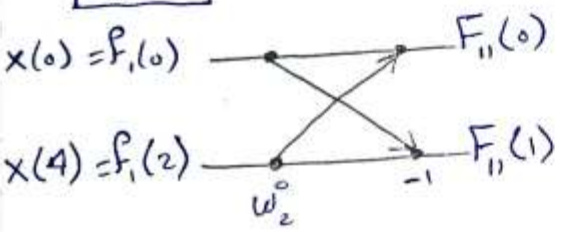
نجزأ الفردى و الزوجى

$$f_1(n) = \{x(0), x(2), x(4), x(6)\}$$

$$f_{11}(n) = \{x(0), x(4)\}$$
$f_1(0)$ $f_1(2)$

"even numbered"

$F_{11}(K)$ ⇒ 2-Point DFT



$$f_{12}(n) = \{x(2), x(6)\}$$
$f_1(1)$ $f_1(3)$

"odd numbered"

$F_{12}(K)$ ⇓

for $F_{11}(K)$, $F_{12}(K)$

$$F_1(K) = F_{11}(K) + W_{N/2}^{K} F_{12}(K)$$

$$F_1(K + \tfrac{N}{2}) = F_{11}(K) - W_{N/2}^{K} F_{12}(K)$$

$$K = 0 \Rightarrow F_1(0) = F_{11}(0) + W_4^0 F_{12}(0)$$
$$K = 1 \Rightarrow F_1(1) = F_{11}(1) + W_4^1 F_{12}(1)$$
} for eq. 1

$$K = 2^0 \Rightarrow F_1(2) = F_{11}(0) - W_4^0 F_{12}(0)$$
$$K = 1 \Rightarrow F_1(3) = F_{11}(0) - W_4^1 F_{12}(1)$$
} for eq. 2

ثانی تقسیم لـ $x(n)$

$$F_2(n) = \{x(1), x(3), x(5), x(7)\}$$

$$F_{21} = \{x(1), x(5)\}$$

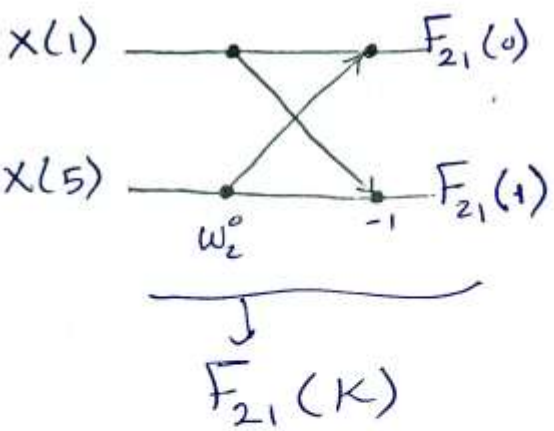
$F_2(0)$ $F_2(2)$

"even numbered"

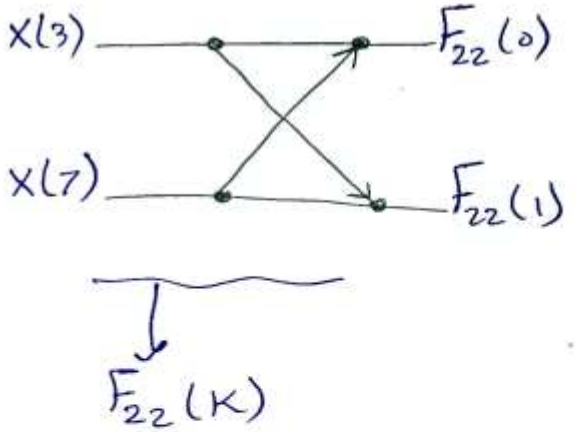
$$F_{22}(n) = \{x(3), x(7)\}$$

$F_2(1)$ $F_2(3)$

"odd numbered"



$F_{21}(K)$



$F_{22}(K)$

① $F_2(K) = F_{21}(K) + W_{N/2}^{K} F_{22}(K)$ → ③

② $F_2(K + \frac{N}{2}) = F_{21}(K) - W_{N/2}^{K} F_{22}(K)$ → ④

$K=0 \Rightarrow F_2(0) = F_{21}(0) + W_4^{\circ} F_{22}(0)$

$K=1 \Rightarrow F_2(1) = F_{21}(1) + W_4^{1} F_{22}(1)$

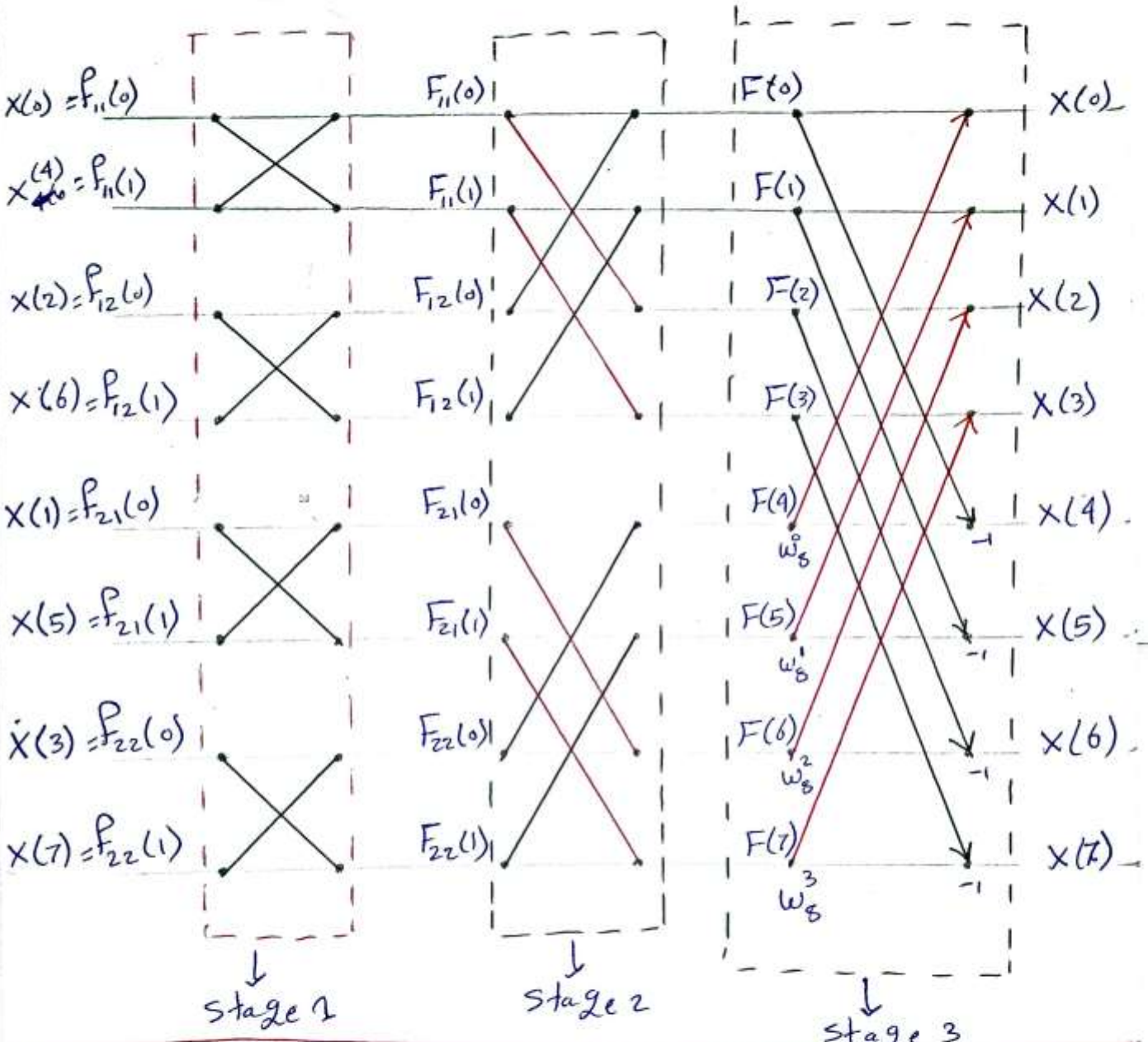
} → eq. ③

$K=0 \Rightarrow F_2(3) = F_{21}(0) - W_4^{\circ} F_{22}(0)$

$K=1 \Rightarrow F_2(4) = F_{21}(1) - W_4^{1} F_{22}(1)$

} → eq. ④

Page 4 — "Butterfly" باقي رسمة الـ



$x(n) = \{x(0), x(1), \dots x(7)\}$

ممكن تحسب الـ Seq بالطريقة دي

$f_1(n) = \{x(0), x(2), x(4), x(6)\}$

$f_2 = \{x(1), x(3), x(5), x(7)\}$

$\{x(0), x(4)\}$ $\{x(2), x(6)\}$

$\{x(1), x(5)\}$ $\{x(3), x(7)\}$

← طريقة أخرى لمعرفة الـ Seq. بعكس أرقام الـ Binary

0	0	0	0
1	0	0	1
2	0	1	0
3	0	1	1
4	1	0	0
5	1	0	1
6	1	1	0
7	1	1	1

عكس!

0	0	0	0
1	0	0	4
0	1	0	2
1	1	0	6
0	0	1	1
1	0	1	5
0	1	1	3
1	1	1	7

Notes

① No. of stages $= V = \log_2 N$

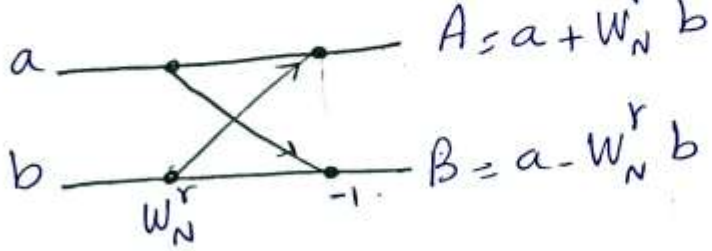
ex $N = 4 \Rightarrow V = \log_2 4 = 2$

$N = 8 \Rightarrow V = \log_2 8 = 3$

② No. of butterflies for each stage $= \frac{N}{2}$

Total No. of butterflies $= \frac{N}{2} \log_2 N$

③



no. of Complex multiplication = 1

no. of Complex addition = 2

* Total no. of Complex Multiplication $= \frac{N}{2} \log_2 N$

* Total " of Complex Additions $= 2 * \frac{N}{2} \log_2 N$

$= N \log_2 N$

مقارنة Page 7

FFT Complex Computation	Direct FFT
من رقم (2) في الصفحة السابقة	$X(K) = \sum_{n=0}^{N-1} K(n) W_N^{Kn}$
	$n = 0, 1, \ldots, N-1$
	$K = 0, 1, \ldots, N-1$
* Total no. of Complex Multiplication $= \frac{N}{2} \log_2 N$	* Total no. of Complex multiplication $= N * N$
* Total no. of Complex additions $= N \log_2 N$	* Total no. of Complex additions $= N * (N-1)$

N	Radix-2 DIT FFT	Direct FFT	
4	4	16	no. of Complex Mul.
	8	12	no. of Complex Add.
8	12	64	no. of Complex Mul.
	24	56	no. of Complex Add.

← كلما زادت N اتضح الفرق أكثر.

16	32	240	mul.
	64	256	Add
256	1024	65280	mul.
	2048	62536	Add.

$$W_8^0 = 1$$

$$W_8^1 = \frac{1}{2} - J\frac{1}{\sqrt{2}}$$

$$W_8^2 = -J$$

$$W_8^3 = \frac{-1}{\sqrt{2}} - J\frac{1}{\sqrt{2}}$$

$$\Rightarrow W_N = e^{-J\frac{2\pi}{N}}$$

ex $x(n) = \{1, 1, 1, 1, 0, 1, 1, 1\}$

$X(K) = \{7, 1, -1, 1, -1, 1, -1, 1\}$

جربها على الرسم